Understanding the Small Scales in Rotating Stratified Flow

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We present two significant new results from studies of the multiscale dynamics of rotating and stratified flows: 1) new parameterization of turbulence in the limit of strong rotation and stratification; 2) a systematic way to improve upon the Quasi-Geostrophic (QG) approximation. The QG approximation is commonly used to describe mesoscales in the atmosphere and oceans but cannot capture coupling between wave motions and coherent structures (jets, fronts, vortices and layers) on long time scales. Our work highlights the importance of proper parameterization of fast dynamics for accurate simulation of long-time weather prediction, ocean circulation and climate change.

Parameterization of the small scales in rotating and stratified flows:

An important goal for realistic computations of multiscale systems such as the ocean, atmosphere or climate is accurate parameterization of complex physics in terms of simplified representations, for example linear equations with coefficients determined from theory, experiments and/or observations. Such parameterizations can then be incorporated into models. For example, in meteorological models, large-scale weather patterns evolve based on parameterization of small-scale turbulence.

A common approximation for rotating stratified flows is the so-called quasi-geostrophic approximation. The two- and three-dimensional quasi-geostrophic (2DQG and 3DQG) equations can be derived by filtering all wave motions from their 'parent' equations, respectively the Rotating Shallow Water (RSW) equations and the Boussinesq equations. The 2DQG and 3DQG equations provide a simplified framework for the study of large-scale, slowly varying atmospheric and oceanic dynamics in isolation from small-scale, rapidly varying fluid motions. Such study is often convenient but over-simplified because real physical systems such as the ocean and atmosphere exhibit strong coupling between the fast waves and the coherent large scales (jets, fronts, vortices and layers) over long time scales.

We obtain a new theoretical prediction for the distribution of kinetic and potential energy in the turbulent small scales of rapidly rotating and stratified flow, without assuming quasi-geostrophy. We find that kinetic energy is suppressed in the small, horizontal scales while potential energy is suppressed in the small, vertical scales. These constraints arise from studying the statistics of an important conserved quantity known as the potential enstrophy. Our results provide proper parameterization of small scales which are too expensive to calculate explicitly in realistic simulations of ocean/atmosphere dynamics.

Capturing cyclone-anticyclone asymmetry

Neither the 2DQG nor 3DQG equations can produce strong asymmetry between cyclones and anticyclones as is ubiquitous in geophysical flows, for example, the long-lived anticyclone known as Jupiter's Red Spot, the predominance of cyclones in the mid-latitude North Atlantic, and the fact that hurricanes are always cyclonic.

We have developed a novel approach to derive a hierarchy of models which successively include more and more effects of the wave motions inherent in the full RSW and Boussinesq systems. In sharp contrast to most previous efforts to improve upon QG models, our technique does not rely on a small parameter. This means that our new 'corrections' to the 2DQG and 3DQG model equations need not be small and are likely to capture essential physics over a wide range of rotation rates and stratification strengths.

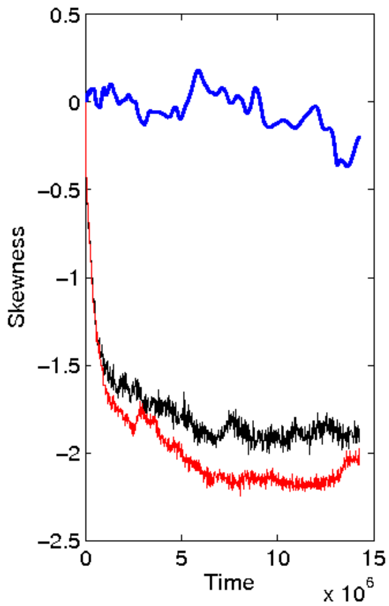

Figure 1: Vorticity skewness vs time during decay from random initial conditions: full Rotating Shallow Water equations (red); 2DQG (blue); new PPG model including QG interactions and all interactions involving one gravity wave (black). Negative skewness indicates the predominance of anticyclonic vortices.

To illustrate the power of our new models, we compare decay dynamics of the full RSW equations, the 2DQG model equation filtering all wave motions, and a new model denoted PPG. The PPG model improves upon 2DQG by adding all nonlinear interactions involving one gravity wave. Figure 1 shows the evolution of the vorticity skewness during decay from random initial conditions, where negative skewness indicates the formation of anticyclonic vortices. One can see that the full RSW and PPG equations quickly develop a negative skewness, whereas the skewness associated with the 2DQG models remains close to zero indicating roughly equal numbers of cyclones and anticyclones.

Our approach is based on a Fourier (wave) representation of the original RSW and Boussinesq equations, for which there are 27 types of nonlinear interactions (some redundant). Retaining a larger subset of interactions results in a model with more complexity. In physical space, the new models can be used with realistic boundary conditions, and they naturally conserve energy. The clear advantage over previous models is lack of dependence on a small parameter.

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